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## Part I. Background information about sea waves.

**KEY WORDS:** *Waves types. Group wave speed. Phase wave speed. Deep water. Wind waves. Swell. Wave energy. Wave spectrum. Significant wave height. Wave generation.*

*Fully developed sea.*

### Sea waves classification.

Sea waves are mechanical energy that has been transferred from wind, earthquakes, landslides, or other waves to the sea water. Sea waves can be classified in various ways (see the Table below). One classification uses the forces, which generate the waves. In ascending order of wave lengths we have:

- Meteorological forcing (wind, air pressure); wind sea and swell belong to this category.
- Earthquakes; they generate tsunamis, which are shallow water or long waves.
- Tides (astronomical forcing); they are always shallow water or long waves.

WAVE	PERIOD	WAVELENGTH	CAUSE	WAVE TYPE
Capillary	< 0.1 sec	< 2 cm	Local winds	Deep to shallow
Wind Waves	1-10 sec	1-10 m	Local winds	Deep or shallow
Swell	10-30 sec	Up to hundreds of km	Distant storm	Shallow or intermediate
Seiche	10 min-10 hr	Up to hundreds of km	Wind, tsunamis	Shallow or intermediate
Tsunami	10-60 min	Up to hundreds of km	Earthquakes under(or near) ocean	Shallow or intermediate
Tide	12.4 - 24.8 hr	Up to thousands of km	Gravitational attraction of sun and moon	Shallow

**Table 1.** Some of the characteristics of different sea waves

In this lecture we will consider sea waves generated by the meteorological forcing i.e. **wind waves** and **swell**.

### **Wind waves**

Wind waves are the most common form of a class of waves called surface gravity waves. Wind waves are produced by the local prevailing wind. They travel in the direction of the prevailing wind, i.e. a northerly wind will produce southerly moving waves. Wind waves are very irregular in appearance. There are several factors that influence the height of a wave: wind speed, wind duration and fetch (distance over water that the wind is blowing in a single direction.).

### **Swell**

Swell waves are wind-generated waves that have moved away from their area of formation. Swell has some distinct physical characteristics that make it unique and distinguish it from a wind wave. Once energy or generating forces no longer affect the waves, the forced waves become free waves moving at speeds due to their periods and wavelengths. Because swell waves are no longer receiving energy from the wind, their spectrum of frequencies is smaller than that of wind waves. Swell waves are also smoother and more regular in appearance than wind waves.

### **Wave motion**

Sea waves are advancing crests and troughs of water propagated by the force of the wind. When winds start to blow, the frictional effect of the wind on the water creates ripples that form more or less regular arcs of long radii. As the wind continues to blow, the ripples increase in height and become waves. A wave is visible evidence of energy moving in an undulating motion through a medium, such as water. As the energy moves through the water, waves of water do not move horizontally, they only move up and down. Sea surface waves are progressive because the wave form moves (progresses) horizontally from one location to another.

The diagram below shows how water particles move in deep water, we can see that all particles make a circular movement in the same direction. They move up on the wave's leading edge, forward on its crest, down on its trailing slope and backward on its trough. At the surface the diameter of the orbit = the wave height (H). Decreasing energy from the wind with depth -> decrease in diameter of wave orbits. When the depth =  $\frac{1}{2}$  the wavelength ( $D = L/2$ ) there is negligible motion.

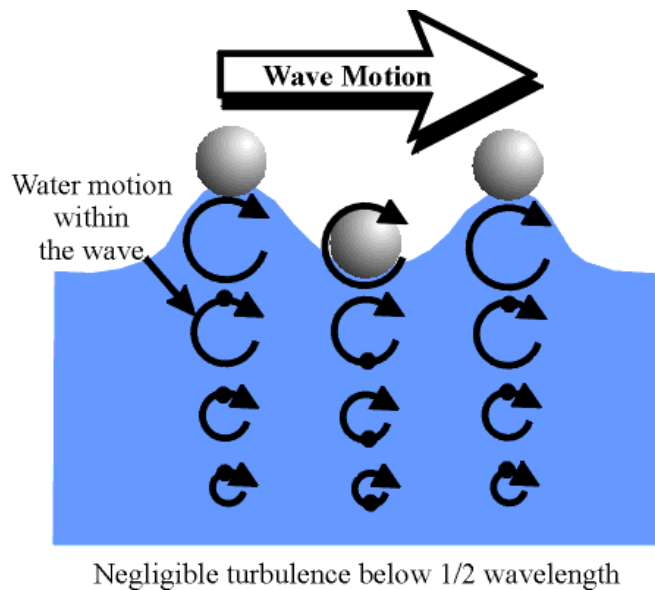


Fig. 1. Deep water wave motion.

As waves move across the sea, only the shape and energy of the wave moves forward; the water particles remain behind.

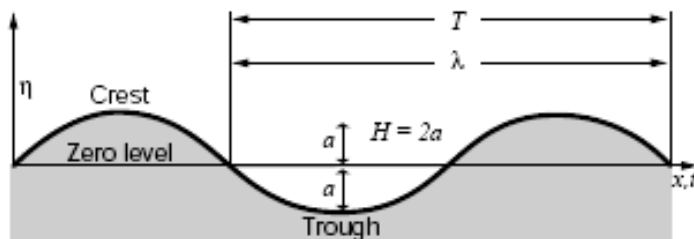


Fig. 2. A simple wave. (adopted from the WMO Guide, 1998)

The ideal sea surface wave, as can be seen above, is sinusoidal. Sine waves are a theoretical concept seldom observed in reality. The portion of the wave that is elevated above the undisturbed sea surface is the **crest**. The portion that is depressed below the surface is the **trough**.

In order to simulate your own sea waves, see the web site:

<http://www.nationalgeographic.com/volvooceanrace/interactives/waves/index.html>

A good illustration of the wave motion is also given on the web site:

<http://www.coastal.udel.edu/faculty/rad/linearplot.html>

All waves have the following properties in common:

- **Amplitude (a).** The amplitude of a wave is the maximum vertical displacement of a particle of the wave from its rest position. In the case of ocean waves, the rest position is sea level.
- **Wave Height (H).** Wave height is the vertical distance from the top of the crest to the bottom of the trough. Wave height is measured in meters.
- **Period (T).** The period of a wave is the time interval between successive wave crests, and it is measured in seconds.
- **Frequency (f).** The frequency of waves is the number of waves passing a given point during 1 second. It is the reciprocal of the period. In general, the lower the frequency, the longer the wave period; the larger the frequency, the shorter the wave period.
- **Wave Length (L).** The wave length is the horizontal distance between two successive crests or from a point on one wave to the corresponding point on the succeeding wave. Wave length is measured in meters.
- **Wave Speed.** There are two speeds used in ocean wave forecasting: individual wave speed and group speed.
- **Wave Speed (or "celerity")** is the speed an individual wave moves through water. The equation below expresses wave speed for all wavelengths and all water depths:

$$C \cong \sqrt{\frac{gL}{2\pi} \tanh\left(\frac{2\pi h}{L}\right)}$$

where:

C = Wave Speed (m/s)

g = gravity = 9.8 (m/s<sup>2</sup>)

L = wavelength (m)

$\pi$  = 3.14159

$\tanh$  = hyperbolic tangent

$h$  = water depth (m)

$2\pi/L$  = wave number

However, the equation can be simplified for deep water or shallow water use. A wave is considered to be a **deep water** wave as long as water depth ( $h$ ) exceeds  $1/2$  the wavelength ( $L$ ). In deep water, the hyperbolic tangent of  $2\pi h/L$  approaches 1. If the  $\tanh$  term in the full speed equation is replaced by 1, the deep water wave speed simplifies to the:

$$C = \sqrt{\frac{gL}{2\pi}} = 2.26 \sqrt{L} = 3.02 T$$

A wave is considered to be a **shallow water** wave as long as water depth ( $h$ ) is less than  $1/20$  the wavelength ( $L$ ). **The shallow water waves and near shore phenomena such as shoaling, refraction and breaking waves will be considered in the special module.**

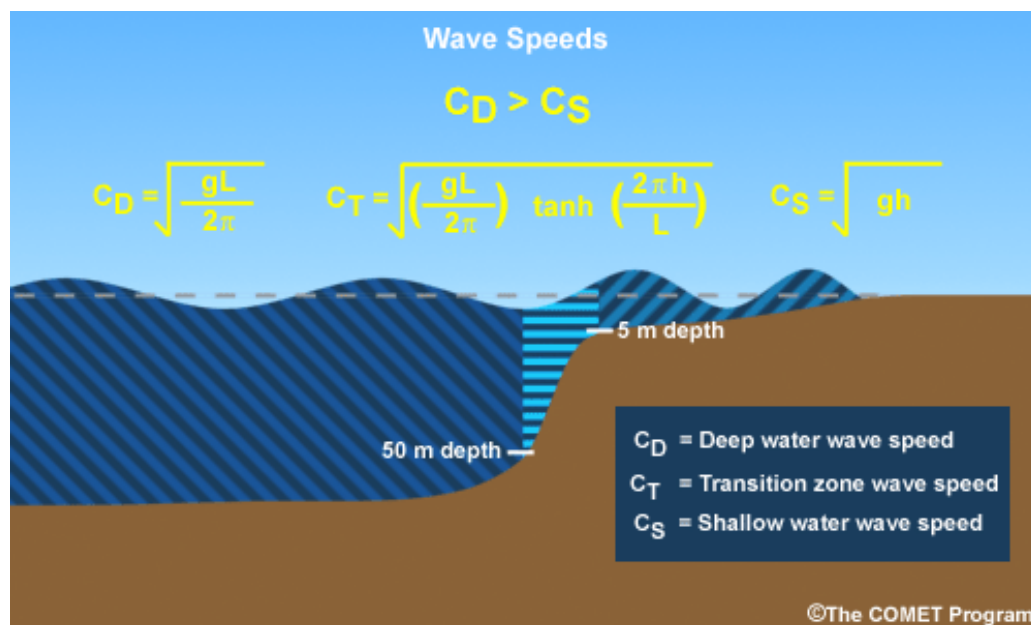


Fig. 3. This figure (adopted from the COMET Program) shows wave speed in deep, transition and shallow water

## Group Velocity

The wave speed equation tells how fast an individual wave moves through the water. However, to forecast the arrival times of waves or swells from some generation area as a whole, group velocity is used. Group velocity for deep water waves equals one-half the individual wave speed:

$$C_g = 0.5C$$

Group Velocity for  
Deep Water Waves

As waves travel away from their fetch generation region, they move in a wave group. This packet of energy advances forward at the group velocity, while individual waves within the group move from the back to the front edge of the wave group at a speed based on their wavelength. Wave group velocity (*not* individual wave speed) must be used to estimate the time it takes for waves/swell to arrive at a location from afar.

**In deep water, group velocity is half the wave phase speed or wave celerity, while in shallow water the group velocity is approximately equal to the individual wave celerity.**

## Useful simple relationships

If the wave period (T) and the wavelength (L) are known, then the wave speed (C) at that point can be determined by this relationship:

**Equation 1:**

$$C = \frac{L}{T}$$

As we've seen in the previous sections, the simplified deep water wave speed equation is dependent only on the wavelength (L). The deep water wave speed can be found by knowing only the wave period. Hence if either the period or wavelength is known, the deep water wave speed can be determined.

**Equation 2:            (deep water only)**

$$C = 1.56 T$$

**Equation 3:            (deep water only)**

$$L = 1.56 T^2$$

In deep water if either the period or the wavelength is known the other can be found, independent of calculating the speed. (The same is not true for shallow water waves.)

**Equation 4:** (shallow water only)

$$L = \sqrt{gh} T$$

The wavelength in shallow water is dependent on the wave period (T) and the water depth (h). However, wave period remains fixed as the wave travels from deep to shallow water. Therefore, the wave period from deep water calculations can be used (if known) along with the water depth to get the wavelength in shallow water.

### **Wave energy.**

We have noted that waves are associated with motion in the water. The wave represents a flow of energy and its present as potential energy due to the change in elevation of the surface water, and kinetic energy due to the motion of the water particles in their orbits.

It is important to note that the energy does **not** move at the same speed as the wave, the phase speed. It moves with the speed of groups of waves rather than individual waves. The total energy of regular sinusoidal waves contained in 1 m<sup>2</sup> of sea depends on the water density, gravity acceleration and the wave height :

$$E = \frac{1}{8} \rho g H^2, \text{ which is the same as: } E = \frac{1}{2} \rho g a^2$$

where:

**E** = wave energy per unit area (J/m<sup>2</sup>)

**ρ** = water density (kg/m<sup>3</sup>)

**g** = gravity (m/s<sup>2</sup>)

**H** = wave height (m)

**a** = wave amplitude (m)

### **Wave Spectrum.**

If we look out to sea, we notice that waves on the sea surface are not simple sinusoids. The surface appears to be composed of random waves of various lengths and periods. We can however, with some simplifications, come close to describing the surface. The simplifications

lead to the concept of the **spectrum  $S(f, \theta)$**  of ocean waves. The wave spectrum is the term that describes mathematically the distribution of wave energy with frequency  $f$  and direction  $\theta$ . Remember that ocean waves are composed of a multitude of sine waves, each having a different frequency. Each sine wave contains a certain amount of energy, and the energy of all the sine waves added together is equal to the **total energy** present in the ocean waves. The total energy present in the ocean waves is not distributed equally throughout the range of frequencies; instead, in every spectrum, the energy is concentrated around a particular frequency ( $f_{max}$ ) that corresponds to a certain wind speed.

It is difficult to work with actual energy values of these sine waves; for this reason the square of the wave amplitude has been substituted for energy. This value is proportional to wave energy.

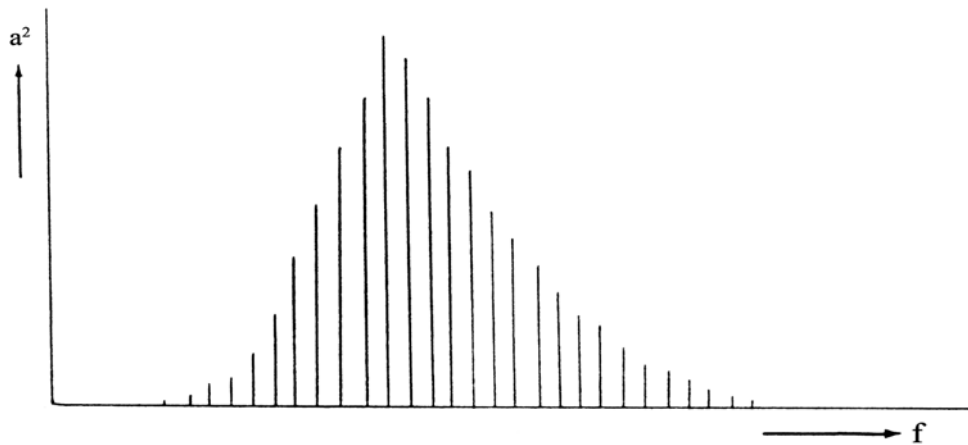


Fig. 4. The square of the wave amplitude plotted against frequency for a single value of wind speed constitutes the spectrum of waves.

Thus, a graph of the spectrum is needed for each wind speed, and the energy associated with each sine wave can be determined from these graphs. Each wind speed produces a particular spectrum; and the higher the wind speed, the larger the spectrum.

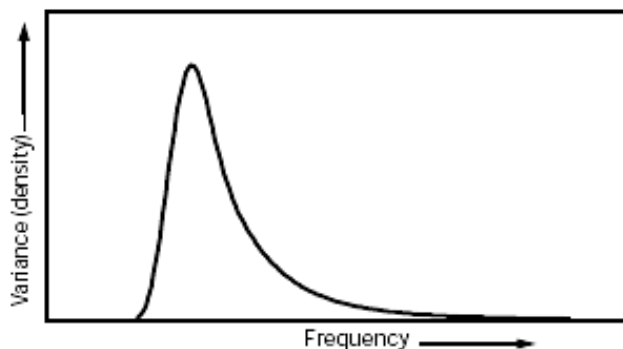


Fig.5. Typical wave-variance spectrum for a single system of wind waves.



The total energy in a sea is represented by the area under the spectral plot. This total energy is a measure of the energy of all waves of any size that make up the 'sea'; (remember the 'sea' is made up of many component waves) Predominate wave in a sea is represented by the period of the peak of the spectrum ( $T_p$ ).

The **Pierson-Moskowitz spectrum** (Pierson and Moskowitz, 1964) is often used as a model spectrum for a fully developed sea, an idealized equilibrium state reached when duration and fetch are unlimited. In its original form, the PM model spectrum is:

$$S(f) = \alpha g^2 (2\pi)^{-4} f^{-5} \exp\left\{-0.74\left[\frac{g}{2\pi U f}\right]^4\right\}$$

Where:  $S(f)$  – energy density ( $m^2 s^{-1}$ );  $f$  - frequency (Hz);  $U$ - wind speed ( $ms^{-1}$ );  $\alpha = 8.1 \times 10^{-3}$ .

The peak frequency for the PM spectrum is:

$$f_p = 0.877 \frac{g}{2\pi U} \text{ (Hz)}$$

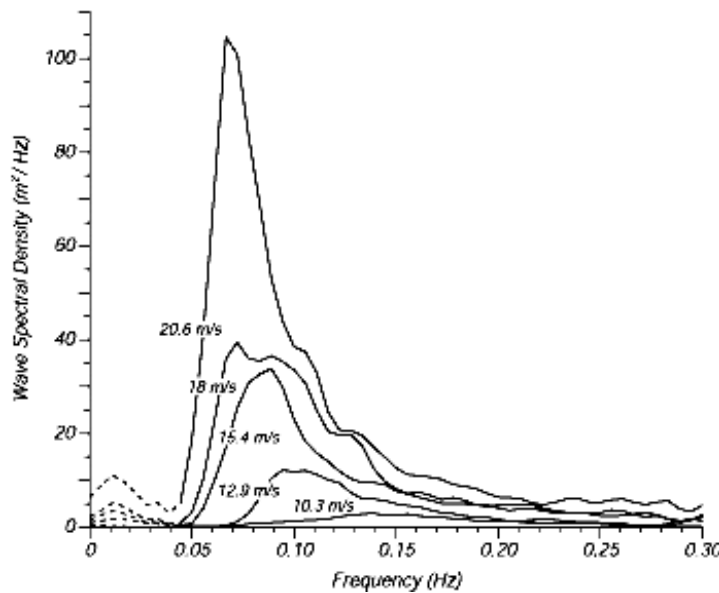


Fig. 6. Wave spectra of a fully developed sea for different wind speeds according to Moskowitz (1964). (after <http://oceanworld.tamu.edu>)

The **JONSWAP spectrum** is often used to describe waves in a growing phase. Hasselmann *et al.*, (1973), after analyzing data collected during the Joint North Sea Wave Observation Project JONSWAP, found that the wave spectrum is never fully developed. It continues to develop through **non-linear, wave-wave interactions** even for very long times and distances. The basic form of the spectrum is in terms of the peak frequency rather than the wind speed:

$$S(f) = \alpha g^2 (2\pi)^{-4} f^{-5} \exp\{-1.25 (f/f_p)^{-4}\}$$

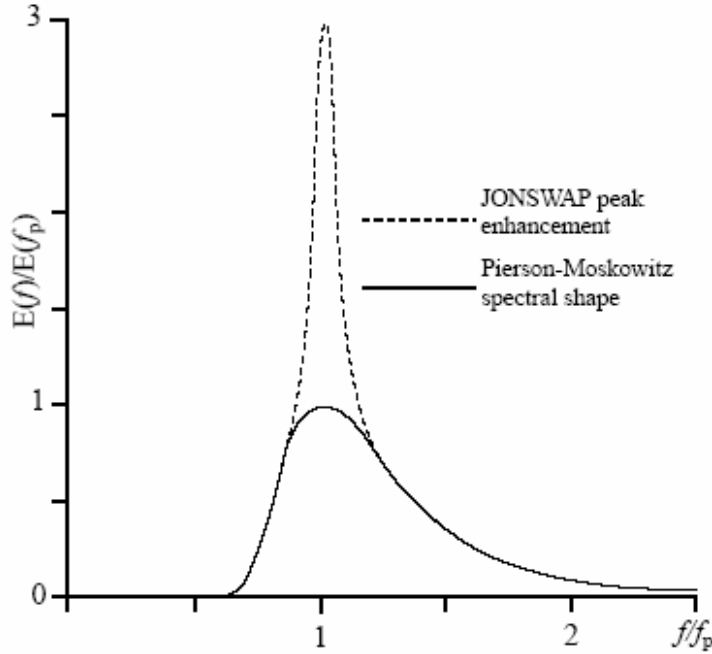


Fig. 7. General form of a JONSWAP spectrum as a function of  $f/f_p$

The JONSWAP spectrum is similar to the Pierson-Moskowitz spectrum except that waves continue to grow with distance (or time) and the peak in the spectrum is more pronounced. The latter turns out to be particularly important because it leads to enhanced **non-linear interactions** and a spectrum that changes in time according to the theory of Hasselmann (1966). **Non-linear wave-wave interactions** cause the redistribution of energy within the wave spectrum. There is a dynamic balance between energy entering the wave field because of wind input and energy leaving the wave field because of nonlinear fluxes to higher frequencies. As energy is fed into the waves from the wind, it is redistributed through nonlinear wave-wave interaction. Energy is transferred from the peak of the spectrum to lower frequencies (decreasing the peak frequency or increasing the peak period) and to high frequencies (where it is dissipated).

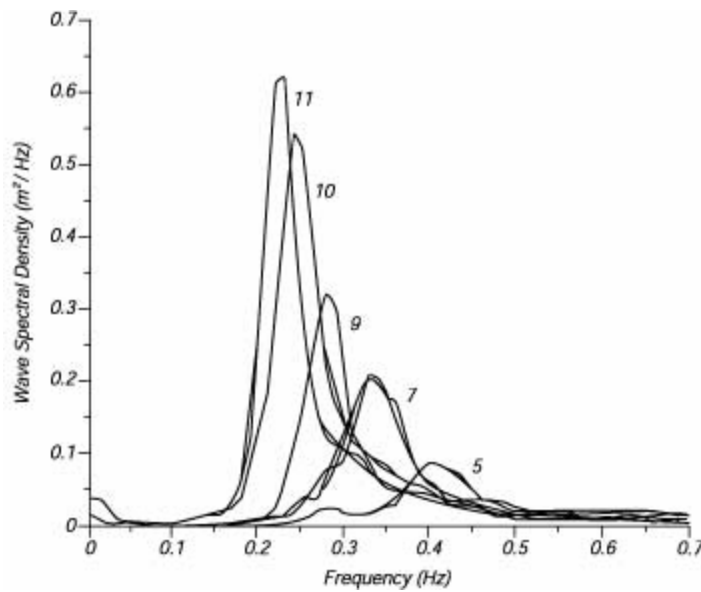


Fig.8. Wave spectra of a developing sea for different fetches according to Hasselmann et al., (1973). (after <http://oceanworld.tamu.edu> )

As the fetch increases, the spectral peaks shift toward longer periods and the total amount of wave energy (the area under the curve) increases, but the spectra will not continue to grow at fetches longer than the minimum required for full development.

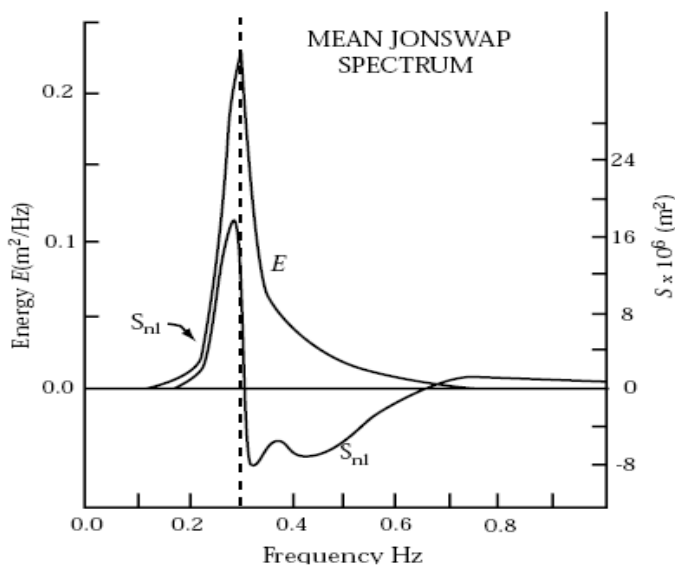


Fig. 9. Growth from non-linear interactions as a function of frequency for the mean JONSWAP spectrum ( $E$ ) (after WMO Guide, 1998)

## Height Classifications

Several symbolic conventions are used in marine meteorology to classify wave heights within the wave spectrum.

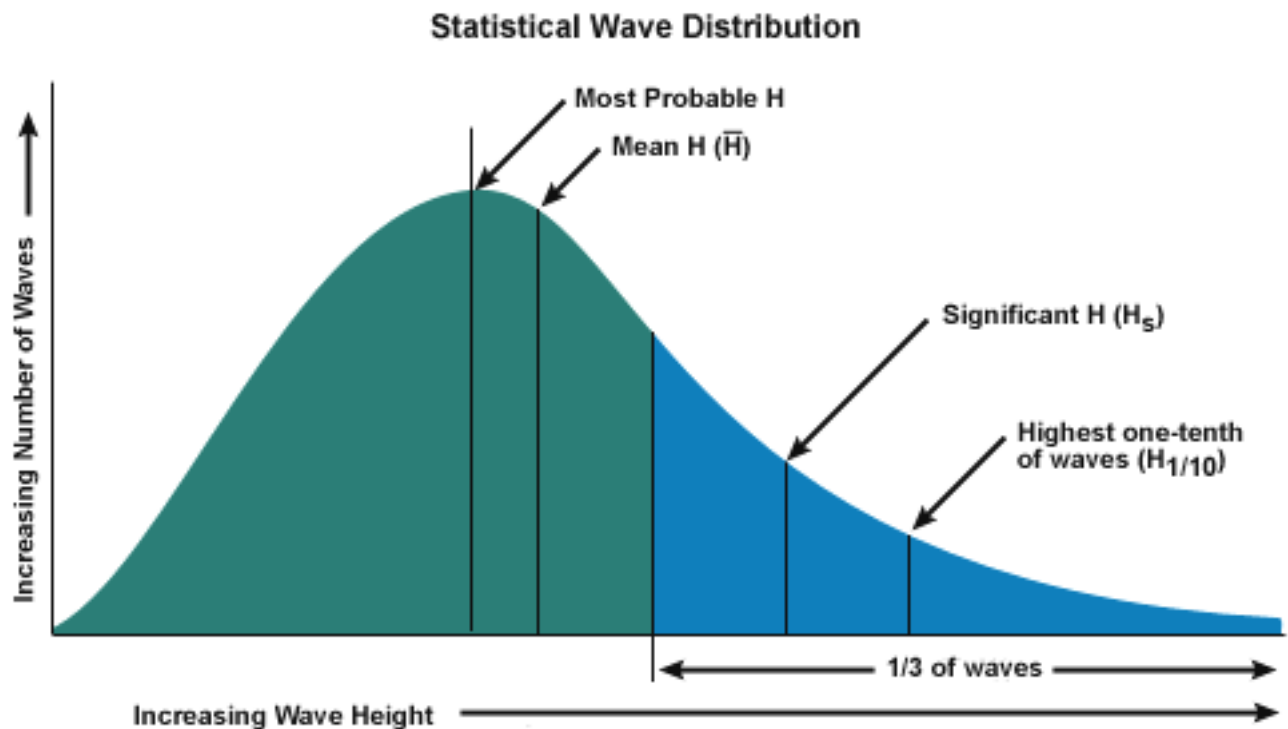


Fig.10. Wave heights classification (adopted from the COMET program)

where:

Mean H = Average wave height

**$H_s$  or  $H_{1/3}$  = Significant wave height**

$H_{1/10}$  = Highest one-tenth wave height

$H_{max}$  = Maximum probable wave height for a large sample of waves.

$H_s$  is the mean height of the highest one third of the waves passing a point.  $H_s$  is of particular interest, since it is generally accepted that visual observations of wave height tend to approximate to the significant wave height.

The averaged periods of the waves used to compute significant wave height is known as the significant wave period. Other wave height statistics can be computed from  $H_s$ , the standard used because nearly all subjective wave heights are estimated in terms of  $H_s$ :

$$H_{1/10} = 1.27H_s$$

$$H_{1/100} = 1.67H_s$$

$$H_{max} = 2.0H_s, \text{ for a reasonably large sample of waves } (> 2000)$$

As a forecaster, you seek to forecast "significant wave height" and "peak wave period"; however, keep in mind that *all* wave groups consist of an entire spectrum of wave heights and periods, **significant wave height and period being only part of this spectrum.**

### Significant wave height derived from the spectrum

A wave spectrum is the distribution of wave energy (or variance of the sea surface) over frequency (or wavelength or frequency and direction, etc.). Thus, as a statistical distribution, many of the parameters derived from the spectrum parallel similar parameters from any statistical distribution.

*(The first **moment** of a distribution of  $N$  observations  $X_1, X_2, \dots, X_n$  is defined as the average of the deviations  $x_1, x_2, \dots, x_n$  from the given value  $X_0$ . The second moment is the average of the squares of the deviations about  $X_0$ ; the third moment is the average of the cubes of the deviations, and so forth. When  $X_0$  is the mean of all observations, the first moment is obviously zero, the second moment is then known as the "variance" of  $X$  and its square root is termed the "standard deviation".)*

Hence, the form of a wave spectrum is usually expressed in terms of the **moments** of the distribution (spectrum). The  $n$ th-order moment,  $m_n$ , of the spectrum is defined by:

$$m_n = \int_0^\infty f^n E(f) df$$

in finite form this is:

$$m_n = \sum_{i=1}^N f_i^n \frac{a_i^2}{2}.$$

From the definition of  $m_n$  it follows that the moment of zero-order,  $m_0$ , represents the area under the spectral curve. In finite form this is:

$$m_0 = \sum_{i=0}^N \frac{a_i^2}{2} = \frac{a^2}{2},$$

which is the total variance of the wave record obtained by the sum of the variances of the individual spectral components. The area under the spectral curve therefore has a physical meaning, which is used in practical applications for the definition of wave-height parameters derived from the spectrum. Recalling that for a simple wave (per unit area),  $E$ , was related to the wave height by:

$$E = \frac{1}{8} \rho_w g H^2,$$

then, if one replaces the actual sea state by a single sinusoidal wave having the same energy, its equivalent height would be given by: the so-called *root-mean-square wave height*.

$$H_{rms} = \sqrt{\frac{8E}{\rho_w g}},$$

$E$  now represents the total energy (per unit area) of the sea state.

We would like a parameter derived from the spectrum and corresponding as closely as possible to the significant wave height  $H_{1/3}$  (as derived directly from the wave record) and, equally, the characteristic wave height  $H_c$  (as observed visually). It has been shown that  $H_{rms}$  should be multiplied by the factor  $\sqrt{2}$  in order to arrive at the required value. Thus, the spectral wave height parameter commonly used can be calculated from the measured area,  $m_0$ , under the spectral curve as follows:

$$H_{m0} = \sqrt{2} \sqrt{\frac{8E}{\rho_w g}} = 4\sqrt{m_0}.$$

$$Hm0 = 1.05 Hs$$

**The additional definitions that the wave forecaster should be familiar are as follows:**

**Fetch (F).** An area of the sea surface over which a wind with a constant direction and speed is blowing, and generating sea waves.

**Duration time (t).** The time that the wind has been in contact with the waves within a fetch.

**Wind field.** A term that refers to the fetch dimensions, wind duration, and wind speed, collectively.

#### **Wave interaction.**

Waves that escape, or outrun a storm are no longer receiving energy from the storm winds and tend to flatten out slightly and the crests become more rounded. As they move out across the ocean, they are likely to meet other trains of swell moving out and away from other storm centers. When two wave trains meet, they pass through each other and continue on. Wave trains may intersect at any angle, and many possible sea surface patterns may result. If 2 trains intersect sharply a checkerboard pattern will be formed, and in some cases two or more trains may phase together so they suddenly develop large waves unrelated to any storms that may become so high they break losing some of their energy.

#### **Deep-water waves.**

When depth  $d >$  or equal to  $1/2 L$ , ocean waves are unaffected by water depth. The diameter of the orbital paths of water particles under these waves decreases as depth increases below the surface, and shrinks to zero at a  $d = 1/2 L$ .

In **deep water**, the wavelength is equal to the acceleration due to gravity ( $g$ ) divided by  $2\pi$  times the square of the wave period ( $T$ ). The value of the earth's gravity ( $g$ ) is  $9.81 \text{ m/sec}^2$

The deep-water wavelength (in meters) is calculated with:

$$L_0 = \frac{g}{2\pi} T^2 = 1.56 T^2$$

#### **Shallow Water waves.**

When  $d < \text{or equal to } 1/20 L$ , ocean waves are only under the control of water depth, and the orbits are elongated ellipses with a major axis in the horizontal direction and the minor axis in the vertical direction. Only the minor axis decreases as depth below the surface increases, so that near the bottom, the water motion is only horizontal and moves back and forth with the passage of a wave. Wave celerity is directly proportional to depth, as shown below. This means that as water depth decreases, waves slow down.

The shallow water wavelength is calculated with:

$$L = \frac{gT^2}{2\pi} \tanh \frac{2\pi h}{L}$$

**The shallow water waves and near shore phenomena such as shoaling, refraction and breaking waves will be considered in the special part of the web-based materials.**

### **Wave dispersion.**

This sorting of waves by wavelength is called **wave dispersion**. The speed of deep-water waves directly depends on the wavelength of the waves. We say that deep-water waves show dispersion. **A waves with a longer wavelength travels at higher speed**. In other words, the longest waves outrun the shorter waves and order is created out of chaos.

In contrast, shallow-water waves show no dispersion. Their speed is independent of their wavelength. It depends, however, on the depth of the water.

### **Wave steepness**

There is a maximum height for any given wavelength. This value is determined by the ratio of the wave's height to the wave's length and is the measure of steepness of the wave. The wave steepness in deep water is defined as:  **$S=H/L$**

According to wave theory, when  $H/L > 1/7$ , waves become too steep and unstable, so they break ; therefore,  $H/L > 1/7$  is called the "breaking criteria". In a many-component wave sea, however, H and L are statistical parameters calculated from the spectra; so even though they still obey the breaking criteria, the definition is slightly different. Waves generated within the fetch area typically are very steep, nearly approaching and



occasionally exceeding the one-seventh breaking threshold as seen by wave white-capping. As waves move away from the wave generation region, wave steepness decreases.

### Growth of wind waves

Three factors affect the growth of wind waves. First, the **wind speed** must be blowing faster than the transfer of energy from wave crest to wave crest. The second factor is the amount of time the wind blows, or **wind duration**. The third factor is the uninterrupted distance over which the wind blows without a change in direction, or the **fetch**. Any one of these factors can limit the wave height. If the wind speed is low, it doesn't matter how far and how long the wind blows over the water, no large waves will be produced. If the wind speed is great but short, again, no large waves will be formed and strong wind over a short area will also not produce large waves.

Most waves at sea are progressive wind waves. They are build up by the wind, restored by gravity and travel in a particular direction. As waves are being generated, they are forced to get larger by the input of energy forced waves. Due to variations in the winds of the storm area, energy at different intensities is transferred to the sea surface at different rates, resulting in waves with a variety of periods and heights. Once a wave is created with its period, the period doesn't change. The speed may change but the period remains the same. The period is a constant property of the wave until the wave is lost by breaking at sea, through friction, or crashing against the shore. Generally high wind speeds of a long duration produce large waves with long wavelengths and wave periods.

The term, a **fully-developed sea** is the largest wave size theoretically possible for a specific **wind speed, wind duration, and fetch**. Any longer exposure to winds at the same wind strength will not increase the size of the waves.

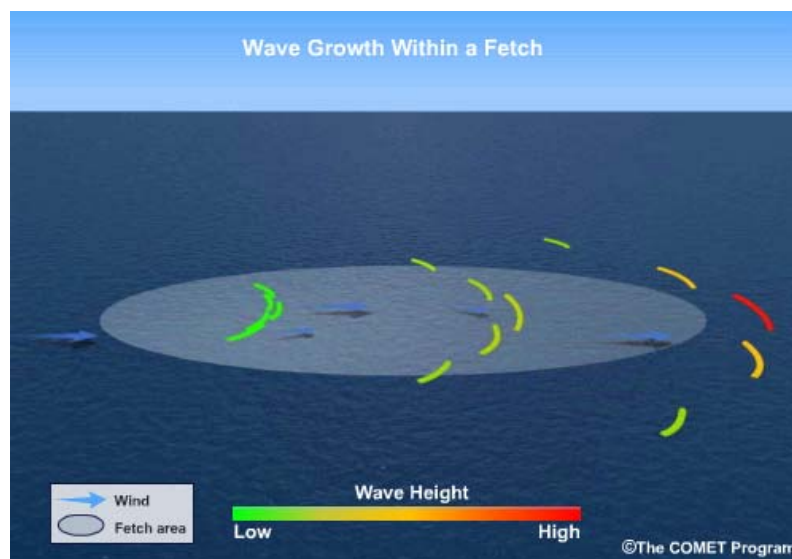


Fig. 11. Wave growth within a fetch (adopted from the COMET Program).

## **Whitecaps**

Even after the sea are fully developed, the wind may continue to transfer momentum to the ocean but, because the waves cannot grow any larger (they are fully developed), the excess energy supplied by the wind must be dissipated. This dissipation occurs when the waves break and, through the turbulent dissipation of energy, "whitecaps" are created.

**Angular spreading.** Angular spreading results from waves traveling radially outward from the generating area rather than in straight lines or banks because of different wind direction in the fetch. Although all waves are subject to angular spreading, the effect of such spreading is compensated for only with swell waves because the spreading effect is negligible for wind waves still in the generating area. Angular spreading dissipates energy.